Найти производные функций, используя правила и формулы дифференцирования:

1. $$\left(\sqrt[4]{2x^3+x+1}\right)'=\left((2x^3+x+1)^{\frac{1}{4}}\right)'=\frac{1}{4}(2x^3+x+1)^{-\frac{3}{4}}(2x^3+x+1)'=\frac{2(x^3)'+x'+1'}{4\sqrt[4]{(2x^3+x+1)^3}}=$$
$$=\frac{2\cdot 3x^2+1+0}{4\sqrt[4]{(2x^3+x+1)^3}}=\frac{6x^2+1}{4\sqrt[4]{(2x^3+x+1)^3}}.$$

2. $$(\ln(3x^2+x-1))'=\frac{(3x^2+x-1)'}{3x^2+x-1}=\frac{3(x^2)'+x'-1'}{3x^2+x-1}=\frac{3\cdot 2x+1-0}{3x^2+x-1}=\frac{6x+1}{3x^2+x-1}.$$

3. $$\left(\frac{x^2+1}{x^3-1}\right)'=\frac{(x^2+1)'(x^3-1)-(x^2+1)(x^3-1)'}{(x^3-1)^2}=\frac{((x^2)'+1')(x^3-1)-(x^2+1)((x^3)'-1')}{(x^3-1)^2}=$$
$$=\frac{(2x+0)(x^3-1)-(x^2+1)(3x^2-0)}{(x^3-1)^2}=\frac{2x(x^3-1)-(x^2+1)3x^2}{(x^3-1)^2}.$$

4. $$\left(\frac{1-\mathrm{tg}(3x)}{1+\mathrm{tg}(3x)}\right)'=\frac{(1-\mathrm{tg}(3x))'(1+\mathrm{tg}(3x))-(1-\mathrm{tg}(3x))(1+\mathrm{tg}(3x))'}{(1+\mathrm{tg}(3x))^2}=$$
$$=\frac{(1'-(\mathrm{tg}(3x))')(1+\mathrm{tg}(3x))-(1-\mathrm{tg}(3x))(1'+(\mathrm{tg}(3x))')}{(1+\mathrm{tg}(3x))^2}=$$
$$=\frac{-\frac{(3x)'}{\cos^2(3x)}(1+\mathrm{tg}(3x))-(1-\mathrm{tg}(3x))\frac{(3x)'}{\cos^2(3x)}}{(1+\mathrm{tg}(3x))^2}=\frac{\frac{(3x)'}{\cos^2(3x)}(-1-\mathrm{tg}(3x)-1+\mathrm{tg}(3x))}{(1+\mathrm{tg}(3x))^2}=$$
$$=\frac{\frac{3}{\cos^2(3x)}(-2)}{(1+\mathrm{tg}(3x))^2}=\frac{-6}{\cos^2(3x)\cdot(1+\mathrm{tg}(3x))^2}=\frac{-6}{(\cos(3x)\cdot(1+\mathrm{tg}(3x)))^2}=\frac{-6}{(\cos(3x)+\sin(3x))^2}.$$

5. $$\left(\frac{1}{\arccos(1-x)}\right)'=-\frac{(\arccos(1-x))'}{(\arccos(1-x))^2}=+\frac{(1-x)'}{(\arccos(1-x))^2\sqrt{1-(1-x)^2}}=$$
$$=\frac{-1}{(\arccos(1-x))^2\sqrt{1-(1-2x+x^2)}}=\frac{-1}{(\arccos(1-x))^2\sqrt{2x-x^2}}.$$

6. $$\left(e+(\mathrm{tg}(4x))^{\pi}\right)'=e'+\left((\mathrm{tg}(4x))^{\pi}\right)'=0+\pi(\mathrm{tg}(4x))^{\pi-1}(\mathrm{tg}(4x))'=$$
$$=\pi(\mathrm{tg}(4x))^{\pi-1}\frac{(4x)'}{\cos^2(4x)}=\frac{4\pi(\mathrm{tg}(4x))^{\pi-1}}{\cos^2(4x)}.$$

7. $$((\mathrm{ctg}(5x))^3)'=3(\mathrm{ctg}(5x))^2(\mathrm{ctg}(5x))'=-3(\mathrm{ctg}(5x))^2\frac{(5x)'}{\sin^2(5x)}=\frac{-3\cdot 5\mathrm{ctg}^2(5x)}{\sin^2(5x)}=\frac{-15\mathrm{ctg}^2(5x)}{\sin^2(5x)}$$

8. $$\left(\frac{1+(3x+1)\lg x}{1+\lg x}\right)'=\frac{(1+(3x+1)\lg x)'(1+\lg x)-(1+(3x+1)\lg x)(1+\lg x)'}{(1+\lg x)^2}=$$

$$=\frac{(1'+((3x+1)\lg x)')(1+\lg x)-(1+(3x+1)\lg x)(1'+(\lg x)')}{(1+\lg x)^2}=$$

$$=\frac{((3x+1)'\lg x+(3x+1)(\lg x)')(1+\lg x)-(1+(3x+1)\lg x)\left(0+\frac{1}{x\cdot\ln 10}\right)}{(1+\lg x)^2}=$$

$$=\frac{\left(3\lg x+(3x+1)\frac{1}{x\cdot\ln 10}\right)(1+\lg x)-(1+(3x+1)\lg x)\frac{1}{x\cdot\ln 10}}{(1+\lg x)^2}.$$

9. $\left(x\cdot\sqrt{1-x^2}\cdot\sin x\right)'=x'\cdot\sqrt{1-x^2}\cdot\sin x+x\cdot\left(\sqrt{1-x^2}\right)'\cdot\sin x+x\cdot\sqrt{1-x^2}\cdot(\sin x)'=$

$$=1\cdot\sqrt{1-x^2}\cdot\sin x+x\cdot\frac{(1-x^2)'}{2\sqrt{1-x^2}}\cdot\sin x+x\cdot\sqrt{1-x^2}\cdot\cos x=$$

$$=\sqrt{1-x^2}\cdot\sin x+x\cdot\frac{-2x}{2\sqrt{1-x^2}}\cdot\sin x+x\cdot\sqrt{1-x^2}\cdot\cos x=$$

$$=\sqrt{1-x^2}\cdot\sin x-\frac{x^2\cdot\sin x}{\sqrt{1-x^2}}+x\cdot\sqrt{1-x^2}\cdot\cos x.$$

10. $\left((\cos(2x-1))^{1-x^2}\right)'=\left(e^{\ln(\cos(2x-1))^{1-x^2}}\right)'=\left(e^{(1-x^2)\ln(\cos(2x-1))}\right)'=$

$$=e^{(1-x^2)\ln(\cos(2x-1))}\left((1-x^2)\ln(\cos(2x-1))\right)'=$$

$$=e^{\ln(\cos(2x-1))^{1-x^2}}\left((1-x^2)'\ln(\cos(2x-1))+(1-x^2)\left(\ln(\cos(2x-1))\right)'\right)=$$

$$=(\cos(2x-1))^{1-x^2}\left((1'-(x^2)')\ln(\cos(2x-1))+(1-x^2)\frac{(\cos(2x-1))'}{\cos(2x-1)}\right)=$$

$$=(\cos(2x-1))^{1-x^2}\left((0-2x)\ln(\cos(2x-1))+(1-x^2)\frac{-\sin(2x-1)(2x-1)'}{\cos(2x-1)}\right)=$$

$$=(\cos(2x-1))^{1-x^2}\left(-2x\cdot\ln(\cos(2x-1))-(1-x^2)\cdot\operatorname{tg}(2x-1)\cdot 2\right).$$

11. $\left((\sin x)^{1+\operatorname{tg}x}\right)'=\left(e^{\ln(\sin x)^{1+\operatorname{tg}x}}\right)'=\left(e^{(1+\operatorname{tg}x)\ln(\sin x)}\right)'=e^{(1+\operatorname{tg}x)\ln(\sin x)}((1+\operatorname{tg}x)\ln(\sin x))'=$

$$=e^{\ln(\sin x)^{1+\operatorname{tg}x}}((1+\operatorname{tg}x)'\ln(\sin x)+(1+\operatorname{tg}x)(\ln(\sin x))')=$$

$$=(\sin x)^{1+\operatorname{tg}x}\left((1'+(\operatorname{tg}x)')\ln(\sin x)+(1+\operatorname{tg}x)\frac{(\sin x)'}{\sin x}\right)=(\sin x)^{1+\operatorname{tg}x}\left(\frac{\ln(\sin x)}{\cos^2 x}+(1+\operatorname{tg}x)\frac{\cos x}{\sin x}\right).$$

12. $\left(\left(\operatorname{ctg}3^{-x}\right)^{\ln(5x+\sqrt{x})}\right)'=\left(e^{\ln\left(\operatorname{ctg}3^{-x}\right)^{\ln(5x+\sqrt{x})}}\right)'=\left(e^{\ln(5x+\sqrt{x})\ln\left(\operatorname{ctg}3^{-x}\right)}\right)'=$

$$=e^{\ln(5x+\sqrt{x})\ln\left(\operatorname{ctg}3^{-x}\right)}\left(\ln(5x+\sqrt{x})\ln\left(\operatorname{ctg}3^{-x}\right)\right)'=$$

$$=e^{\ln\left(\operatorname{ctg}3^{-x}\right)^{\ln(5x+\sqrt{x})}}\left(\left(\ln(5x+\sqrt{x})\right)'\ln\left(\operatorname{ctg}3^{-x}\right)+\ln(5x+\sqrt{x})\left(\ln\left(\operatorname{ctg}3^{-x}\right)\right)'\right)=$$

$$=\left(\operatorname{ctg}3^{-x}\right)^{\ln(5x+\sqrt{x})}\left(\frac{(5x+\sqrt{x})'}{5x+\sqrt{x}}\ln\left(\operatorname{ctg}3^{-x}\right)+\ln(5x+\sqrt{x})\frac{(\operatorname{ctg}3^{-x})'}{\operatorname{ctg}3^{-x}}\right)=$$

$$=\left(\operatorname{ctg}3^{-x}\right)^{\ln(5x+\sqrt{x})}\left(\frac{(5x)'+(\sqrt{x})'}{5x+\sqrt{x}}\ln\left(\operatorname{ctg}3^{-x}\right)-\ln(5x+\sqrt{x})\frac{(3^{-x})'}{\operatorname{ctg}3^{-x}\cdot\sin^2 3^{-x}}\right)=$$

$$=\left(\operatorname{ctg}3^{-x}\right)^{\ln(5x+\sqrt{x})}\left(\frac{5x'+\frac{1}{2\sqrt{x}}}{5x+\sqrt{x}}\ln\left(\operatorname{ctg}3^{-x}\right)-\ln(5x+\sqrt{x})\frac{3^{-x}\cdot\ln 3\cdot(-x)'}{\cos 3^{-x}\cdot\sin 3^{-x}}\right)=$$

$$=\left(\operatorname{ctg}3^{-x}\right)^{\ln(5x+\sqrt{x})}\left(\frac{5+\frac{1}{2\sqrt{x}}}{5x+\sqrt{x}}\ln\left(\operatorname{ctg}3^{-x}\right)+\ln(5x+\sqrt{x})\frac{3^{-x}\cdot\ln 3}{\cos 3^{-x}\cdot\sin 3^{-x}}\right).$$

13. $\left(\ln 2+2\sin\left(\sqrt{x}^{x\sin 2x}\right)\right)'=(\ln 2)'+2\left(\sin\left(\sqrt{x}^{x\sin 2x}\right)\right)'=0+2\cos\left(\sqrt{x}^{x\sin 2x}\right)\cdot\left(\sqrt{x}^{x\sin 2x}\right)'=$

$$=2\cos\left(\sqrt{x}^{x\sin 2x}\right)\cdot\left(x^{\frac{x\sin 2x}{2}}\right)'=2\cos\left(\sqrt{x}^{x\sin 2x}\right)\cdot\left(e^{\ln x^{\frac{x\sin 2x}{2}}}\right)'=2\cos\left(\sqrt{x}^{x\sin 2x}\right)\cdot\left(e^{\frac{x\sin 2x}{2}\ln x}\right)'=$$

$$=2\cos\left(\sqrt{x}^{x\sin 2x}\right)\cdot\left(e^{\frac{x\sin 2x}{2}\ln x}\right)\left(\frac{x\cdot\sin 2x\cdot\ln x}{2}\right)'=$$

$$=2\cos\left(\sqrt{x}^{x\sin 2x}\right)\cdot\left(e^{\ln x^{\frac{x\sin 2x}{2}}}\right)\cdot\frac{1}{2}\cdot\left(x'\cdot\sin 2x\cdot\ln x+x\cdot(\sin 2x)'\cdot\ln x+x\cdot\sin 2x\cdot(\ln x)'\right)=$$

$$=\cos\left(\sqrt{x}^{x\sin 2x}\right)\cdot\left(x^{\frac{x\sin 2x}{2}}\right)\cdot\left(1\cdot\sin 2x\cdot\ln x+x\cdot\cos 2x\cdot(2x)'\cdot\ln x+x\cdot\sin 2x\cdot\frac{1}{x}\right)=$$

$$=\cos\left(\sqrt{x}^{x\sin 2x}\right)\cdot\left(\sqrt{x}^{x\sin 2x}\right)\cdot\left(\sin 2x\cdot\ln x+x\cdot\cos 2x\cdot 2\cdot\ln x+\sin 2x\right).$$

14. Найти вторую производную функции $f(x)=\arcsin\left(\sqrt{x}\right)$.

$$f'(x)=\left(\arcsin\left(\sqrt{x}\right)\right)'=\frac{\left(\sqrt{x}\right)'}{\sqrt{1-\left(\sqrt{x}\right)^2}}=\frac{1}{2\sqrt{x}\sqrt{1-x}}=\frac{1}{2\sqrt{x-x^2}}.$$

$$f''(x)=\left(\frac{1}{2\sqrt{x-x^2}}\right)'=\frac{1}{2}\left(\left(x-x^2\right)^{-\frac{1}{2}}\right)'=-\frac{1}{2}\cdot\frac{1}{2}\left(x-x^2\right)^{-\frac{3}{2}}\left(x-x^2\right)'=-\frac{1-2x}{4\sqrt{\left(x-x^2\right)^3}}=\frac{2x-1}{4\sqrt{\left(x-x^2\right)^3}}.$$